Spectroscopic imaging of the secondary star in AM Her

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ABSTRACT
Spectroscopic observations of AM Her are used to determine the orbital velocity of the secondary star. We describe how the radial velocities and flux deficits derived from the Na\textsc{i} doublet around 8190 \AA\ can be used to map the Na\textsc{i} surface distribution and hence give an estimate of $K_z$, corrected for the effects of irradiation. The resulting surface maps are consistent with the accretion geometry proposed by Cropper and, from the ‘quality’ of the fit, suggest an inclination of 50° or more, higher than that normally quoted.

Key words: binaries: spectroscopic – stars: fundamental parameters – stars: individual: AM Her – novae, cataclysmic variables.
2.2 Finding the surface map

The next step is to construct a surface irradiation map that is able to fit these residual velocities. This work uses the simulation program by J. S. Martin (Martin 1988; Martin et al. 1989) to produce model radial velocities. Originally, the program was designed to calculate the radial velocity curve for a given irradiation model. With the surface mapping approach, we instead look for the Na i doublet line strength distribution that is consistent with the observations, and only later consider what irradiation model could reproduce the observed map.

Because we are trying to calculate a 2D surface map from a 1D velocity curve, it is necessary to make some assumption about the surface distribution. As a simple first approach, we adopted a ‘one-spot’ model, where we assume that there is one region on the star that is being heated and that it is the position and size of this region that is to be found. Because the radial velocity data will typically constrain the equatorial distribution much better than the latitudinal variation (e.g. Collier Cameron 1992), we adopt a simple sin \( \theta \) dependence for the variation with latitude, and use the model to determine the longitudinal variation. This tacitly assumes symmetry about the orbital plane, which will certainly not be correct if, as expected, the accretion region (the assumed source of the irradiation) is out of the orbital plane. However, if the bulk of the accretion luminosity arises close to the white dwarf surface, this symmetry will be relatively small, and is unlikely to be resolvable using the Na i doublet, which has a fairly large intrinsic width. The addition of an extra parameter, to determine the latitude of the centre of the spot, would probably only degrade the entire fit, and make the correction to the radial velocity amplitude less reliable.

Although it gives no clue to the real latitudinal variations, the model does readily give both any longitudinal asymmetry about the L\(_1\) point and the size of the affected area. It is also simple and requires only three fitting parameters, which allows rapid convergence to the best fit. At present the radial velocity data are not accurate enough to pick out small spots on the surface, so the model attempts only to locate the one main area of irradiation. We have chosen the Na i doublet as the line with which to map the surface irradiation, because Brett & Smith (1993) have shown that the Na i flux deficit decreases with increasing irradiation; the sodium is ionized by the incident fluid.

The relative strength of the Na i doublet over the surface is parametrized by

\[
I(\theta, \phi) = \begin{cases} 
I_0 - A \sin \theta \cos \frac{\phi - \phi_0}{2B} & \frac{\phi - \phi_0}{2B} < \frac{\pi}{2} \\
I_0, & \text{otherwise}
\end{cases}
\]

where \( \theta \) and \( \phi \) are the standard spherical polar and azimuthal angles. The angle \( \theta \) is measured from the positive z axis, and \( \phi \) is measured anticlockwise from the positive x axis in the xy plane. (Note: the positive x axis is towards the primary star. Also, the equatorial angle \( \phi \) is related to the orbital phase \( \varphi \) via \( \phi = \pi - 2\pi \varphi \).) Here \( I_0 \) represents the unperturbed flux deficit of the Na i doublet, \( A \) (which is positive) denotes the maximum decrease in the flux deficit on the surface of the secondary star, \( B \) is the equatorial extent of the spot as a fraction of the circumference of the star, and \( \phi_0 \) is the angle measured anticlockwise from the L\(_1\) point to the centre of the heated region. When \( I(\theta, \phi) \) is less than zero, then it is set to zero (this can happen if \( A \) is larger than \( I_0 \)). The Na i doublet is weakest at \( \phi = \phi_0, \theta = \pi/2 \), which is supposed to be the place where the irradiation is strongest.

The choice of a cos \( \phi \) term to represent the longitudinal variation about this point is also justified on grounds of simplicity; given the distorted shape of the secondary star, it is certainly only a rough representation of reality, especially if \( \phi_0 \neq 0 \). We believe that a more sophisticated representation would not yield a significantly more reliable map, given the quality of the data. The agreement of our simple model with the maximum entropy map given in Section 3.3 is some justification for this point of view.

For a given observational phase and inclination, a total spectral profile is generated by adding the Na i doublet profiles from each surface element \( ij \), allowing for the radial velocity (Doppler shift) of the element and weighting each by its apparent area and flux deficit \( I_0 \). For fuller details see Davey & Smith (1992) and Davey (1994).

The coefficients \( A, B \) and \( \phi_0 \) are initially guessed, and a synthetic residual radial velocity curve, \( V_{res}(\phi) \), is obtained for the flux deficit distribution \( I(\theta, \phi) \). The same orbital phases as the original observations are used, and the goodness-of-fit between the synthetic residual velocities and the observed residual velocities is calculated using the reduced \( \chi^2 \) statistic

\[
\chi^2 = \frac{1}{N-3} \sum_{k=1}^{N} \left[ \frac{V_{res}(k) - V_{obs}(k)}{\sigma_{res}(k)} \right]^2.
\]

Here \( \sigma_{res}(k) \) is the rms deviation for each observation \( k \) at orbital phase \( \varphi(k) \). Since this is not known in general, the values of \( \sigma_{res}(k) \) are taken to be the same, usually \( \sigma \) from the elliptical fit, for all phases. The fitting coefficients are suitably corrected until the smallest value of \( \chi^2 \), i.e., the best fit, is reached and the convergence stops. At this stage one needs to determine whether the value of \( K_j \) initially chosen was correct.

The observed eccentricity is now compared to the eccentricity of the model radial velocities. The value of \( K_j \) is used is then decreased if the model eccentricity is too low and increased if it is too high. For this new value of \( K_j \), a new self-consistent value of \( q \) (if the period, \( M_2 \) and inclination are assumed fixed) has to be calculated, and \( A, B \) and \( \phi_0 \) are again computed to minimize \( \chi^2 \). Correction of \( K_j \) proceeds in this iterative manner, with usually only two or three iterations required, until one achieves results consistent with the observations.

The errors for the model eccentricity were computed from an elliptical fit to the model radial velocities. This also gives the error in the semi-amplitude of the elliptical fit to the model radial velocities, which when combined with the observed error on \( K_{obs} \) (which is usually much larger) gives an error for the corrected \( K_j \) value. The error on \( \phi_0 \) is obtained by fixing \( \phi_0 \) at various values around the optimal value and reminizing \( \chi^2 \) with just \( A \) and \( B \) allowed to vary. A plot of \( \chi^2 \) against \( \phi_0 \) is shown in Fig. 1 for AM Her. A 1σ
confidence level corresponds to $\chi^2_{\text{min}} + 1/(N - 2)$ (where $N$ is the number of observations and there are only two free parameters now); $2\sigma$ corresponds to $\chi^2_{\text{min}} + 4/(N - 2)$, etc. (Press et al. 1992).

2.3 Surface mapping from flux deficits

Wade & Horne (1988) observed in Z Cha that the Na i doublet and TiO flux deficits varied with phase, so one can also use the observed variation of the sodium doublet line strength to put further constraints on the surface irradiation map.

The Na i doublet flux deficit is derived from the original spectra using

$$fd(\text{Na} i) = \int \frac{8215}{8165} [c(\lambda) - f(\lambda)] \, d\lambda,$$  \hspace{1cm} (4)

where $f(\lambda)$ is the profile of the Na i doublet, and $c(\lambda)$ is the mean continuum level. The continua were fitted using the routine POLFIT (written by T. R. Marsh) which fits a fifth-order polynomial to the data; having obtained the rms deviation for the fit, the routine then rejects any points that lie more than 2.5 times the rms value away from the fit. As an example, a co-added spectrum of AM Her generated from six spectra around phase 0.0 is shown in Fig. 2 with its continuum fit. The spectrum was fitted only in the region 7850 to 8300 Å, because we were interested only in the region around the sodium doublet. Also, the data points marked by an 'x' were excluded from the fit.

If good flux deficit data are available for any system, then the surface distribution can be mapped using the same method as that used to model the residual radial velocities. Ideally, this requires spectrophotometric data, whereas the radial velocity maps can be made without spectrophotometry. As before, the surface of the secondary star is divided into small elements, and a Na i doublet line strength ascribed to each using the formula given in equation (2). Again the surface map is used to generate synthetic spectra, from which the flux deficit is calculated using equation (4). Now, rather than using the radial velocities, the goodness-of-fit between the model flux deficits and the observed flux deficits is calculated using

$$\chi^2_{\text{fit}} = \frac{1}{N - 3} \sum_{k=1}^{N} \left[ \frac{fd_{\text{obs}}(k) - fd_{\text{mod}}(k)}{\sigma_{\text{obs}}(k)} \right]^2.$$  \hspace{1cm} (5)

Here $\sigma_{\text{obs}}(k)$ is the rms deviation for each observation $k$ at orbital phase $\phi(k)$ and can be estimated from the rms deviation found from the continuum fit to the observed spectra. As before, the coefficients $A, B$ and $\phi_0$ are suitably corrected until the smallest value of $\chi^2$, i.e., the best fit, is achieved and the convergence stops.

From this resulting surface map one can then get predicted radial velocity corrections that should be applied to the observed radial velocities. The advantage of this method is that an initial estimate of $K_2$ for the system is not required. The radial velocity corrections will depend on the orbital inclination, the period and also weakly on the secondary mass and mass ratio, so these might need to be estimated initially. However, the method does give a rough estimate of the inclination of the system when it is not eclipsing, although the symmetry assumptions made in the modelling prevent any precise conclusions.

3 RESULTS FOR AM HER

3.1 Determination of the inclination

Although we have no spectrophotometric measurements for AM Her, the continuum flux (Fig. 3) seems to vary smoothly and consistently over the two nights. The dip in the continuum flux around phase 0.5 could be caused by obscuration of the front face of the secondary star by the gas stream and magnetospheric impact hotspot, or by the geometrical projection of the accretion stream; both explanations suggest an angle of inclination higher than the value of $35^\circ$ usually quoted (Brainerd & Lamb 1985).

The radial velocity and flux deficit data were fitted simultaneously, using the method described in Section 2, with a range of system inclinations to try to estimate the value that gives the best results. These results are shown in Fig. 4 and

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Minimum $\chi^2$ plot for AM Her used for computing the error on $\phi_0$. 


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clearly show that for $i \geq 45^\circ$ the fit improves significantly. Unfortunately, the inclination cannot be determined particularly accurately because the data are equally well fitted for inclinations greater than $45^\circ$ by using a surface line strength distribution that does not decrease so markedly at the front of the star. None the less, despite the assumptions in the modelling, which make firm conclusions about the inclination rather doubtful, the large variation in flux deficit with phase would be hard to understand if the inclination were as low as $35^\circ$. An upper limit on the inclination of about $60^\circ$ to $70^\circ$ can be inferred from the lack of eclipses of the L1 point or gas stream observed around phase zero. So an inclination of between $45^\circ$ and $60^\circ$ would be the preferred value from this work. This is also consistent with $i = 52^\circ$ obtained by Wickramasinghe et al. (1991).

### 3.2 Surface maps

The data were now fitted for an orbital inclination of $50^\circ$, period = 3.094 h, $M = 0.26 M_\odot$ (from the main-sequence assumption) and $q = 0.6$, where $q = M_2/M_1$. These values are in good agreement with the parameter ranges derived by Southwell et al. (1995). The results are plotted in Fig. 5 for the flux deficit data and Fig. 7 for the radial velocity data, and are also summarized in Table 1. The corresponding surface maps are shown in Figs 6 and 8 respectively. The view of the secondary star is from the top looking down the axis of rotation ($z$ axis) with the white dwarf to the right along the $x$ axis. The units on the axes are in terms of the binary separation for this system.

As can be seen in Fig. 6, the resulting surface map from the Na I flux deficit data compares well with that in Fig. 8 obtained from the radial velocity data. Table 1 shows that the model eccentricity is much closer to the observed value of 0.068 ± 0.010 when $i = 50^\circ$ than when $i = 35^\circ$; the values of $\chi^2$ for the maps are also lower for the higher inclination.

The surface maps obtained are relatively unaffected by choosing $K_2$ in the range allowable for consistency with the observed eccentricities. In particular, the phase angle of the terminator does not appear to change significantly as $K_2$ is varied. Since the strength of the Na I doublet is related to the temperature at the surface (Brett & Smith 1993), these plots also give an indication of the temperature distribution over the secondary star.

Here the irradiation is by the white dwarf and accretion column only and might be expected to be almost exactly symmetrical about the L1 point. However, for AM Her the direction of the magnetic pole of the white dwarf, and hence the likely direction of the gas stream, is given as $\beta = 61^\circ$ and $\psi = 31^\circ$ (Cropper 1988), where $\beta$ is the angle measured downwards from the axis of rotation, and $\psi$ is the angle measured anticlockwise from the line joining the white dwarf to the secondary star. This geometry is shown in Fig. 9 and suggests that the incoming accretion column is blocking radiation produced very close to the white dwarf, thereby shielding an area on the leading side of the secondary star. Indeed, observations of GQ Mus (Shaham 1993), also an AM Her type system, show irradiation of the front face of the secondary star. In that system there is a shadow on the red dwarf star around the L1 point (of approximately $40^\circ$ in extent) that is thought to be caused by flux from the accretion column being unable to radiate directly towards the secondary star because of the incoming gas stream.

### 3.3 Maximum-entropy reconstruction of surface features

Another method for identifying features on the surface of the secondary star is maximum-entropy reconstruction (using the MMSYS package of Gull and Skilling; Skilling & Bryan 1984; Skilling & Gull 1985). The surface mapping routine used in this section was written by René Rutten and uses the original spectra directly to map surface features by modelling both the Doppler shifts and the line strength variations around the binary orbit (Rutten 1993; Rutten & Dhillon 1994). The advantage of this technique is that it does not use the measured radial velocities and flux deficits.
Figure 3. Continuum flux results for AM Her. The integrated continuum flux is given by $\int f_\lambda \sin c(\lambda) \, d\lambda$ and is in arbitrary flux units.

Figure 4. Plot of total $\chi^2$ against a range of inclinations for the AM Her system.

Figure 5. Flux deficit results for AM Her in which the model has fitted the flux deficits and an inclination of 50° has been used. The integrated flux deficit is in arbitrary units.

derived from the original spectra, but instead uses the actual variations in the line profile with phase and hence can, in theory, identify features covering only a few per cent of the surface. Indeed, this method has already been used for several years to map star spots on fast rotating magnetic stars (e.g., AB Dor: Collier Cameron & Unruh 1994). It has also been used to image accretion discs in CVs, where it is known as Doppler tomography or Doppler imaging (Marsh & Horne 1988).

The same binary system parameters as before were used
agreement is good. Since no symmetry assumptions are involved in the maximum-entropy reconstruction, this agreement is a retrospective justification of our simple model; the quality of the data is not sufficient to resolve detail. The resolution is also limited (Davey 1994) by the relatively large intrinsic width (about 3 Å) of each component of the Na i doublet.

Figure 6. Surface map of AM Her showing the strength of the Na i doublet corresponding to the flux deficit results shown in Fig. 5. The light region is where the Na i doublet is weak. The view is from the axis of rotation (i.e., the z axis) looking down on to the top of the secondary star.

Figure 8. Surface map showing the strength of the Na i doublet corresponding to the radial velocity results shown in Fig. 7. The light region is where the Na i doublet is weak. The view is as in Fig. 6.

Figure 7. Residual radial velocity results for AM Her in which the model has fitted the radial velocity data and an inclination of 50° has been used.

Table 1. Radial velocity results for AM Her from mapping flux deficit data and residual velocity data.

<table>
<thead>
<tr>
<th></th>
<th>$K_\text{e}$</th>
<th>$q$</th>
<th>$e$</th>
<th>$\chi^2_r$</th>
<th>$\chi^2_f$</th>
<th>$\phi_0$</th>
<th>extent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i = 35^\circ$</td>
<td>174 ± 2</td>
<td>0.37</td>
<td>0.029 ± 0.002</td>
<td>1.80</td>
<td>2.54</td>
<td>+22° ± 4°</td>
<td>46%</td>
</tr>
<tr>
<td>$i = 50^\circ$</td>
<td>179 ± 2</td>
<td>0.60</td>
<td>0.063 ± 0.006</td>
<td>0.66</td>
<td>1.65</td>
<td>+25° ± 4°</td>
<td>35%</td>
</tr>
<tr>
<td>$i = 50^\circ$</td>
<td>179 ± 2</td>
<td>0.60</td>
<td>0.060 ± 0.007</td>
<td>0.95</td>
<td>1.11</td>
<td>+30° ± 5°</td>
<td>29%</td>
</tr>
</tbody>
</table>
Unfortunately, as Rutten & Dhillon (1994) point out, maximum-entropy optimization is a non-linear procedure, and therefore uncertainties in the resulting map cannot be derived directly from uncertainties in the data. A proven method of obtaining errors in the surface map is to employ a Monte Carlo technique, but the large number of reconstructions that one would need to perform would at present require a prohibitively large amount of computing time. With increasing computing power, this will cease to be a problem, and Roche tomography promises to become a powerful tool for studying the surface of the secondary star.

The principal disadvantage of the technique at present, and the reason that we have not adopted it wholesale for our mapping of secondaries, lies in the feature that is also its strength: its use of individual line profiles. Absorption lines from the secondary are generally weak, and the signal-to-noise (S/N) ratio in the lines is relatively low, making the variation in line profile with orbital phase rather noisy. Numerical experiments (Davey 1994) suggest that a S/N ratio of 50 or more is required to make effective use of maximum-entropy mapping; the data presented here for AM Her have a S/N ratio of about 10 and are only capable of resolving the large-scale features that we have modelled with the one-spot model. It therefore seems better with low S/N data to include some restriction in the model, such as our one-spot assumption, since allowing a free choice for the model may throw up low S/N features whose reality is hard to judge.

Obtaining larger S/N ratios is always difficult for CVs, given the intrinsic faintness of the secondary and the need for short exposures to avoid velocity smearing. The orbital variation also requires spectrophotometry for optimal results (as does our use of flux deficits). Further, the maximum-entropy technique is vulnerable to contamination of the line profile by other sources of light; our technique is not quite so vulnerable to this effect, because of the extra assumption about the light distribution. It is our view that, until good error estimates can be obtained for maximum-entropy maps, there is no great advantage in using this technique except for very high-quality data, since the reality of any features other than a simple single spot around the

Figure 9. Sketch of the magnetic cataclysmic variable AM Her, showing the direction of the accretion pole.

Figure 10. Maximum-entropy map for AM Her in which a 3000-K Na I doublet template has been used as the mapping line, and an orbital inclination of 50° has been assumed. The plot shows a 3D view from phase 0.55.
centre of irradiation would be in doubt. Our one-spot model is a computationally simple way of determining the correction to the radial velocity amplitude, which was the original aim of this work.

4 CONCLUSIONS

When studying the red dwarf star in CV systems irradiation can be expected from the white dwarf and accretion flow, and one needs to allow for this in order to get corrected values for the orbital velocity of the secondary star. A failure to correct $K_2$ when necessary can lead to quite large errors in the mass ratio and incorrect masses. However, a correction of $K_2$ solely from the observed eccentricity in the radial velocity curve can sometimes give the wrong results. A better way is to compute the surface line strength map that best fits the radial velocity data, and this also has the advantage of giving an idea of the size and position of surface features. The radial velocity data can also help to constrain system parameters, and for AM Her suggest an inclination of between 45° and 60°.

Obtaining spectrophotometric data is very important, since it provides surface distribution information that is independent of, but clearly related to, the radial velocities derived from the spectra. In the AM Her system the phase-resolved spectra indicate that the system is being irradiated mainly to one side of the L₄ point, because of the geometry of the gas stream and accretion column, as shown in Fig. 9. Wickramasinghe et al. (1991) propose a slightly different gas stream geometry for AM Her in which there are two accretion regions, as sketched in Fig. 11. Their model has one main accretion region above the orbital plane, 140° anticlockwise from the line of centres, which extends about 8° in phase. The second is below the plane, about 170° away from the line of centres and 20° to 30° in extent. Since the positions of the accretion regions are both more than 90° away from the line of centres, there would seem to be no reason why the white dwarf and accretion column would not have a ‘clear view’ of the whole of the front face of the secondary star. This would therefore be unlikely to give the asymmetrical maps of AM Her (e.g., Fig. 6), whereas the magnetic field geometry proposed by Cropper (1988) does seem to be fully consistent. Of course, this conclusion is tentative, since the structure of the accretion stream, especially near the threading region, is complex and uncertain, and the stream itself may be doing the shadowing.

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