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Higgs mechanism and bulk gauge boson masses in the Randall-Sundrum model

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Assuming the breaking of gauge symmetries by the Higgs mechanism, we consider the associated bulk gauge boson masses in the Randall-Sundrum background. With the Higgs field confined on the TeV-brane, the W and Z boson masses are naturally an order of magnitude smaller than their Kaluza-Klein excitation masses. The electroweak precision data require the lowest excited state to lie above about 30 TeV, with fermions on the TeV-brane. This bound is reduced to about 10 TeV if the fermions reside sufficiently close to the Planck-brane. Thus, some tuning of parameters is needed. We also discuss the bulk Higgs case, where the bounds are an order of magnitude smaller.

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It has recently been realized that the large hierarchy between the Planck scale and the electroweak scale could be related to the presence of extra dimensions [1]. An interesting realization of this concept is the Randall-Sundrum model [2]. It relies on the 5-dimensional non-factorizable geometry

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{1}$$

where $\sigma(y)=k|y|$. The 4-dimensional metric is $\eta_{\mu\nu}=\mathrm{diag}(-1,1,1,1)$, k is the AdS curvature, and y denotes the fifth dimension. This metric results from a suitable adjustment of the bulk cosmological constant and the tensions of the two 3-branes which reside at the S_1/Z_2 orbifold fixed points $y=0$, $y=\pi R$. Because of the exponential (''warp'') factor, the effective mass scale on the brane located at $y=\pi R$ is $M_P e^{-\pi kR}$. If $kR\sim 11$ this scale will be $\mathcal{O}(\mathrm{TeV})$, and the brane is referred to as the ''TeV-brane.'' Hence the model can generate an exponential hierarchy of scales from a small extra dimension.

In the setting of Ref. [2] only gravity propagates in the 5d bulk, while the standard model (SM) fields are confined to the TeV-brane. However, since a microscopic derivation is still missing, it is interesting to study other possibilities. Bulk scalar fields were first discussed in Ref. [3]. The consequences of SM gauge bosons in the bulk were studied in Refs. [4,5]. In Ref. [6] the behavior of fermions in the bulk was investigated, and in Ref. [7] the complete SM was put in the bulk. Finally, bulk supersymmetry was considered in Ref. [8].

Bulk gauge fields are necessary if the SM fermions live in the bulk. By localizing the fermions at different positions in the fifth dimension it seems possible to address the questions of fermion mass hierarchy, non-renormalizable operators and proton decay [9,10,8]. New possibilities for baryogenesis may open up if the fermion separation is reduced by thermal correction in the hot early universe [11].

Bulk vector bosons with bulk masses have been considered to some extent in Refs. [5,7]. It was found that the ''zero'' mode acquires a mass comparable to the mass of the ''first'' Kaluza-Klein (KK) excitation, unless the bulk gauge boson mass is extremely fine-tuned [7]. Since gauge boson KK excitations should have masses in the TeV range [4,5], the W-boson mass could only be generated by reintroducing the original hierarchy problem. This suggests [7] that the Higgs boson should be confined to the TeV-brane, i.e. The gauge boson mass arises from the boundary.

In this paper we will investigate this scenario in more detail. We will study the properties of bulk gauge bosons which are related to broken gauge symmetries, i.e. bulk W and Z bosons. We will show that in the case of a TeV-brane Higgs field, the W and Z boson masses are naturally an order of magnitude smaller than the mass of their first KK excitations. We will demonstrate that the W and Z boson mass ratio and $\sin^2\theta_W$ can be successfully reproduced by a moderate tuning of the brane mass parameter. We also discuss constraints from universality of the coupling of the gauge bosons to fermions. In the phenomenologically viable parameter range we recover the 4D relationship between gauge and Higgs boson masses. Contraints arising in the bulk Higgs case are also briefly discussed.

Let us consider the following equation of motion for a U(1) gauge boson A_N of 5-dimensional mass M:

$$\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{MN}g^{RS}F_{NS}) - M^2 g^{RS}A_S = 0, \tag{2}$$

where g_{MN} denotes the 5-dimensional metric. In general, M arises from some Higgs mechanism and consists of bulk and boundary contributions

$$M^2(y) = b^2k^2 + a^2k\,\delta(y-\pi R) + \tilde{a}^2k\,\delta(y), \tag{3}$$

depending on whether the Higgs fields live in the bulk and/or on the branes. The gauge boson masses can be expressed in terms of the parameters of the Higgs potential. For the TeV-brane Higgs field, for instance, we have

$$a^2 = \frac{g_5^2\mu^2}{2\lambda k} = \frac{g_5^2}{2k}v^2, \tag{4}$$

where μ denotes the Higgs boson mass parameter and λ the quartic coupling, both understood as 4D quantities, and g_5 is the 5D gauge coupling. $v=\mu/\sqrt{\lambda}$ is the VEV of the Higgs field.

Using the metric (1) and decomposing the 5D field as

$$A_\mu(x^\mu,y)=\frac{1}{\sqrt{2\pi R}}\sum_n A_\mu^{(n)}(x^\mu)f_n(y), \tag{5}$$

one obtains [4,5]

$$(-\partial_y^2+2\sigma'\partial_y+M^2)f_n=e^{2\sigma}m_n^2 f_n, \tag{6}$$

where m_n are the masses of the Kaluza-Klein excitations $A_\mu^{(n)}$, and $\sigma'=\partial_y\sigma$. (We work in the gauge $A_5=0$ and $\partial_\mu A^\mu=0$.)

Requiring the gauge boson wave function to be even under the Z_2 orbifold transformation, $f_n(-y)=f_n(y)$, one finds [5]

$$f_n=-\frac{e^\sigma}{N_n}\left[J_\alpha\left(\frac{m_n}{k}e^\sigma\right)+\beta_\alpha(m_n)Y_\alpha\left(\frac{m_n}{k}e^\sigma\right)\right], \tag{7}$$

where the order of the Bessel functions is $\alpha=\sqrt{1+b^2}$. The coefficients β_α obey

$$\beta_\alpha(x_n,\tilde{a}^2)=-\frac{\left(-\frac{\tilde{a}^2}{2}+1-\alpha\right)J_\alpha(x_n)+x_nJ_{\alpha-1}(x_n)}{\left(-\frac{\tilde{a}^2}{2}+1-\alpha\right)Y_\alpha(x_n)+x_nY_{\alpha-1}(x_n)}, \tag{8}$$

$$\beta_\alpha(x_n,\tilde{a}^2)=\beta_\alpha(\Omega x_n,-a^2), \tag{9}$$

where we defined the warp factor $\Omega=e^{\pi kR}$, and $x_n=m_n/k$. Note that for non-vanishing boundary mass terms the derivative of f_n becomes discontinuous on the boundaries. The normalization constants N_n are defined such that $(1/\pi R)\int_0^{\pi R}dy f_n^2=1$.

An analogous discussion also holds in the non-Abelian case.

Equations (8), (9) encode the masses of the different KK states. In the limit $m_n\ll k$ and $m_n\Omega\gg k$, one finds [8]

$$m_n\approx\left(n+\frac{\alpha}{2}-\frac{3}{4}\right)\pi k\Omega^{-1}. \tag{10}$$

In this regime the masses of the excited KK states are independent of the boundary mass terms. The bulk mass term enters via α, but its contribution is also suppressed by the warp factor. This is because the excited states are localized at the TeV-brane as a result of the exponential in their wave functions. If the SM fermions live on the TeV-brane, it was found that the masses of the gauge boson KK states should be in the multi TeV range in order to be in agreement with the electroweak precision date [4]. In the case of bulk fermions the corresponding constraints becoming weaker [7,8], reducing to $m_1>0.5$ TeV for fermions on the Planck-brane [5].

Let us now consider m_0, the mass of the lowest lying state. In the case with neither bulk nor boundary mass term, one finds $m_0=0$, and the corresponding (zero mode) wave function is not localized in the extra dimension, $f_0(y)\equiv 1$. If a bulk or boundary mass term is added, the ''zero'' mode picks up a mass, and its wave function displays a y-dependence.

In the case of a bulk mass term $b\sim 1$, one finds $m_0\sim\pi k\Omega^{-1}$, i.e. approximately of the same value as the first excited state in the massless case [7]. Although the bulk mass term is of order M_p, the gauge boson mass does not become Planck-sized, because f_0 is localized at the TeV-brane, where the effective mass scale is small. At first sight this seems encouraging, but it was also shown in Ref. [7] that extreme fine tuning $b\sim\Omega^{-1}$ is necessary in order to bring down the *W*-boson mass, i.e. m_0, from its natural TeV size range to the experimental value. One therefore would have to start with a weak scale bulk mass term, which is nothing but the original fine tuning problem. These results follow from expanding Eqs. (8), (9) in the regime $x_n\ll 1$ and $x_n\Omega\ll 1$. Along the same lines we find that a gauge boson mass term at the Planck-brane has the same implications,

$$x_0^2\approx\frac{\tilde{a}^2}{2\ln\Omega}=\frac{\tilde{a}^2}{2\pi kR}. \tag{11}$$

Since their is no warp factor suppression, only for $\tilde{a}\sim\Omega^{-1}$ is a *W*-mass below the Kaluza-Klein scale possible.

If the Higgs field is on the TeV-brane however, we arrive at a different conclusion. Expanding Eqs. (8), (9) for $x_0\ll 1$ and $\Omega x_0\ll 1$ we find

$$\Omega^2x_0^2\approx\frac{a^2}{2\ln\Omega}\frac{1}{1-\frac{a^2}{4\ln\Omega}\left(\gamma+\ln\frac{x_0}{2}\right)}, \tag{12}$$

where $\gamma\approx 0.5772$, which reduces to

$$\Omega^2x_0^2\approx\frac{a^2}{2\ln\Omega}=\frac{a^2}{2\pi kR},\quad a\ll 1 \tag{13}$$

$$\Omega^2x_0^2\approx\frac{2}{\ln\Omega}=\frac{2}{\pi kR},\quad a\gg 1. \tag{14}$$

Similar to the case of a bulk or Planck-brane mass term, we find a linear relationship between a and x_0 for small values of a. But in contrast to the former, this behavior remains valid up to $a\lesssim 1$, because of the appearance of the warp factor. For $a\gtrsim 1$, x_0 saturates at a value typically an order of magnitude smaller than x_1, which corresponds to the mass of the first excited state. This demonstrates that a Higgs boson at the TeV-brane can, in principle, explain weak gauge boson masses of order 100 GeV, while keeping the KK states in the TeV range [12]. The saturation results from the drop of the wave function near the TeV-brane for large a which diminishes the overlap with the brane mass term.

In Fig. 1 we show Ωx_0 as a function of a for $\Omega=10^{14}$, i.e. $kR=10.26$. For $a\gg 1$ we obtain $\Omega x_0\sim 0.24$. [In the evaluation we numerically solved Eqs. (8), (9), but Eq. (12) would also reproduce the results at a percent accuracy level.] The mass of the first excited KK state depends very weakly on a. In our example we find it rises from $\Omega x_1(a=0)=2.45$ [4,5]

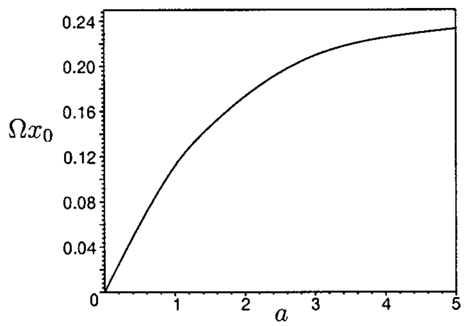

FIG. 1. Ωx_0 versus a for a warp factor $\Omega = 10^{14}$.

to $\Omega x_1(\infty) = 3.88$. In Fig. 2 we display the resulting ratio between the mass of the ground state and the first excited state. For small enough a we find $x_1/x_0 \equiv m_1/m_0 \propto 1/a$, while for large a the ratio approaches $x_1/x \sim 15.4$. For different values of Ω the results hardly change since the warp factor only enters logarithmically in Eqs. (13), (14).

Since the ground state mass scale, i.e. The W and Z boson masses, is experimentally known to be ~ 100 GeV, we conclude that in the brane Higgs scenario $m_1 \gtrsim 1.5$ TeV is necessary. This bound does not rely on the electroweak precision data and is independent of the position of the fermions in the fifth dimension. It could only be weakened if the warp factor in Eq. (14) is substantially reduced, which would reintroduce the hierarchy problem.

We next discuss constraints on KK excitations arising from the electroweak precision data. From Fig. 1 we deduce that the relationship between the boundary mass term a and the ground state mass becomes highly non-linear in the regime $a \gtrsim 1$. As a result the very successful SM prediction that the gauge boson masses are proportional to their couplings to the Higgs boson could be spoiled. We measure the deviation from the linear behavior of $x(a)$ by

$$\delta_1 \equiv \frac{x_0(ar)}{x_0(a)} - r, \tag{15}$$

and take $r = M_W/M_Z = 0.88$. For $\Omega = 10^{14}$ the results are shown in Fig. 3. They are well approximated by $\delta_1 \sim 0.025a^2$, i.e. The non-linearity increases quadratically with a. Since r is measured to an accuracy of about 10^{-3} and no deviations from the SM prediction have been found [13], we require $\delta_1 \lesssim 10^{-3}$. This leads to the modest constraint $a \lesssim 0.2$. From Fig. 2 we deduce that $x_1/x_0 \gtrsim 100$. As a result the mass of the first KK excitation has to obey $m_1 \gtrsim 10$ TeV. The constraint on m_1 is proportional to $1/\sqrt{\delta_1}$. We stress again that it does not depend on where the fermions live. The warp factor only enters logarithmically.

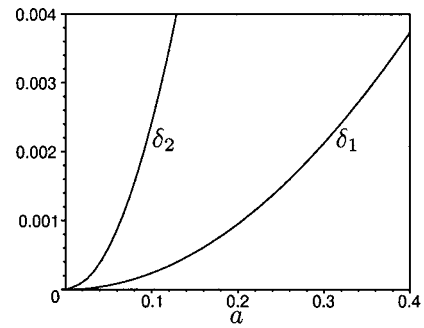

FIG. 3. Plots of δ_1 [see Eq. (15)] and δ_2 (16) versus a, with $\Omega = 10^{14}$.

Once the ''zero'' mode acquires a mass its wave function (7) depends on the y coordinate. In contrast to excited states the wave function tries to avoid the TeV-brane where its mass arises, as shown in Fig. 4. As a result the successful SM predictions of the gauge couplings to fermions of the W and Z bosons can be affected. The resulting constraints depend on the position of the fermions in the fifth dimension.

For example, the coupling of the W boson to a fermion on the TeV-brane is given by $g_0 f_0(\pi R)$, where g_0 denotes the coupling if the boson were massless. Since $f_0(\pi R) < 1$ in the brane Higgs scenario, the resulting gauge coupling is somewhat reduced. In Fig. 3 we present the resulting deviation from the SM prediction,

$$\delta_2 \equiv 1 - f_0(\pi R), \tag{16}$$

as a function of the brane mass parameter a ($\Omega = 10^{14}$). For $\delta_2 \lesssim 10^{-3}$, we find $a \lesssim 0.06$, a constraint more stringent than the one from the mass ratio $r = M_W/M_Z$. From Fig. 2 we learn that $x_1/x_0 \gtrsim 310$, i.e. $m_1 \gtrsim 30$ TeV, a bound which is proportional to $1/\sqrt{\delta_2}$. With this restriction the effects of the KK states are automatically in agreement with the electroweak precision data, which only requires $m_1 \gtrsim 23$ TeV [4].

If the massive gauge boson is coupled to fermions on the Planck-brane the effective gauge couplings hardly deviate

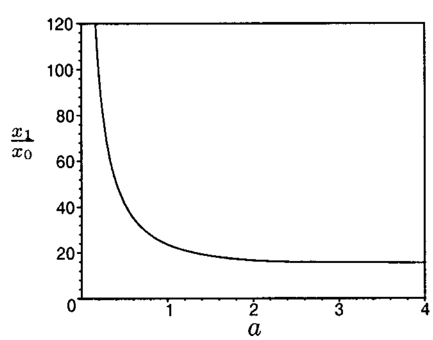

FIG. 2. Mass ratio of the lowest lying and the first excited KK state versus a for $\Omega = 10^{14}$.

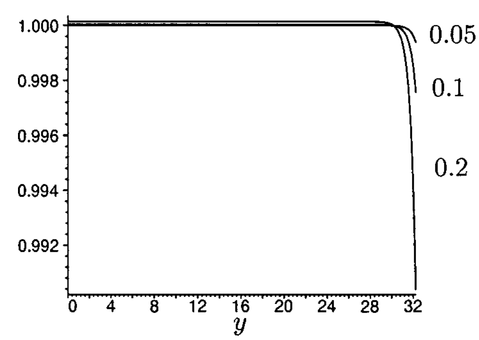

FIG. 4. Ground state wave function $f_0(y)$ [Eq. (7)] for $a = 0.2$, 0.1, 0.05, where $\Omega = 10^{14}$.

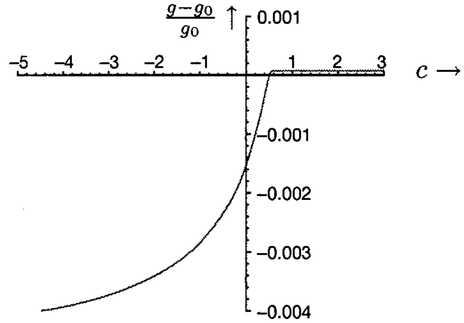

FIG. 5. Deviation of g/g_0 from unity versus the fermion mass parameter c, for $a=0.14$ and $\Omega=10^{14}$.

from the SM prediction, since $f_0(0)\sim 1$ (see Fig. 4). The resulting constraint, $m_1 \gtrsim 4$ TeV, is weaker than that arising from Eq. (15).

Bulk fermions interpolate between the TeV- and the Planck-brane scenarios. As discussed in Refs. [6,8], depending on the bulk mass term $m_\psi = c\sigma'$ for the fermion, the zero mode of the fermion is localized at the TeV-brane ($c<1/2$) or at the Planck-brane ($c>1/2$). For $c=1/2$ the fermionic zero mode is delocalized in the fifth dimension. Since the W-boson wave function has a nontrivial y-dependence, it then couples non-universally to fermions localized at different positions in the fifth dimension. This is completely analogous to the c-dependent coupling of the excited gauge boson states discussed in Ref. [8]. We have repeated the analysis for the ground state of the massive gauge boson. In Fig. 5 we display g/g_0-1 as function of c for $a=0.14$ and $\Omega=10^{14}$, where g_0 would be the coupling of a massless gauge boson. The shape of the gauge coupling of the massive ground state is similar to those of the excites KK states [8]. However, the amplitude of the variation is much smaller. In the limit $c\rightarrow -\infty$, g approaches the result of the TeV-brane fermions (16). In the regime $c \gtrsim 1/2$ the deviation of the SM prediction for g becomes small. In this case a is only constrained by the W,Z boson mass ratio (15).

Taking into account the warp factor, the Higgs boson mass on the TeV-brane is given by [2]

$$M_H = \mu\Omega^{-1} = \sqrt{\lambda} v \Omega^{-1}. \tag{17}$$

In 4D the gauge and Higgs boson masses are related by $M_W^2 = (g^2/2\lambda)M_H^2$. In the brane Higgs scenario this relationship is certainly violated in the regime $a \gtrsim 1$ due to the nonlinearity in $m_0(a)$ (see Fig. 1). However, in the phenomenologically viable parameter range $a \lesssim a_{\mathrm{max}} \approx 0.14$, where $m_0^2 \approx (g_5^2/4\pi R)v^2$ (13), (4), and $g \approx g_5/\sqrt{2\pi R}$, the 4D relation is recovered, up to small correction of order 10^{-3}. Using Eq. (4), the parameters of the Higgs potential have to obey

$$\frac{\mu^2}{k^2} < \frac{a_{\mathrm{max}}^2}{2\pi kR}\frac{2\lambda}{g^2}. \tag{18}$$

Assuming $\lambda \sim 1$ we find that a moderate tuning $\mu \lesssim 0.04k$ is required to reproduce the measured W and Z boson masses in the brane Higgs scenario.

Finally, let us briefly summarize our results for the bulk Higgs case, which may be especially interesting if SM fermions reside on the Planck-brane in order to eliminate unwanted higher dimensional operators. A TeV-brane Higgs case cannot provide masses for these fermions. To solve the hierarchy problem, one has to rely on some additional mechanism, for instance supersymmetry. The W,Z boson mass ratio (15) leads to $m_1 \gtrsim 250$ GeV, which is rather weak compared to the brane Higgs case ($m_1 \gtrsim 30$ TeV). Stronger restrictions arise from the modification of the gauge couplings (16). For Planck-brane fermions we find $m_1 \gtrsim 600$ GeV, while for TeV-brane fermions this increases to $m_1 \gtrsim 3.5$ TeV, bounds which are again weaker than for the TeV-brane Higgs case. The wave function of the ''zero'' mode increases near the TeV-brane. This results also apply in the Planck-brane Higgs scenario.

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